Measuring Conductivity of Copper and Aluminum with Maxwell’s Equations

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Outline

1. **Theory**
   Maxwell’s equations, complex wave vectors and skin depth
2. **Experiment**
   Objective, setup, method for data acquisition
3. **Results**
   Analysis of model fitting, comparison to known values
4. **Conclusions**

Maxwell's Equations

No free charge in conductors

$$\boxed{(i)\nabla \cdot \mathbf{E} = 0} \quad (iii)\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(ii)\nabla \cdot \mathbf{B} = 0 \quad \boxed{(iv)\nabla \times \mathbf{B} = \mu\epsilon\frac{\partial \mathbf{E}}{\partial t} + \mu\sigma\mathbf{E}}$$

Electric and magnetic fields are affected by material conductance

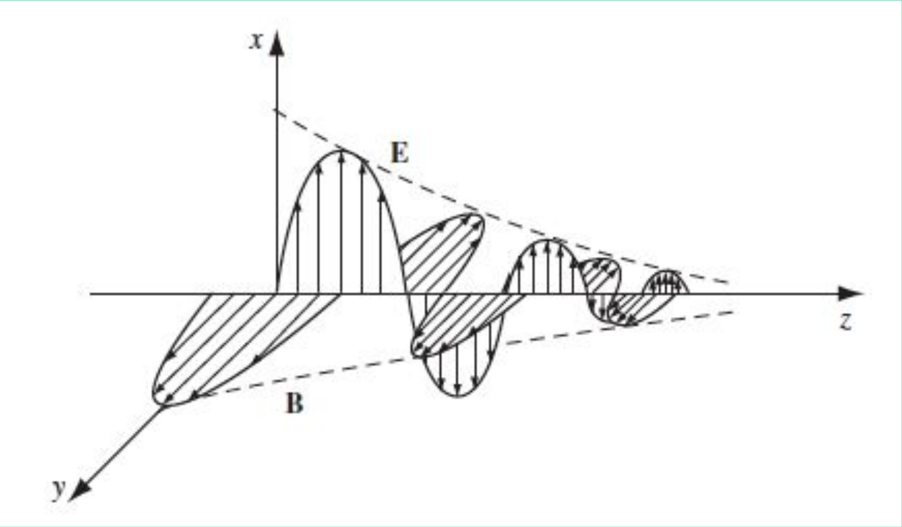
With these two equations, we can solve for our fields **E** and **B**

Complex Wave Vectors

Solving the wave equation, we get plane waves that have complex wave vectors

$$\begin{cases} \tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(\tilde{k}z-\omega t)} \\ \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(\tilde{k}z-\omega t)} \end{cases}$$

$$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$$

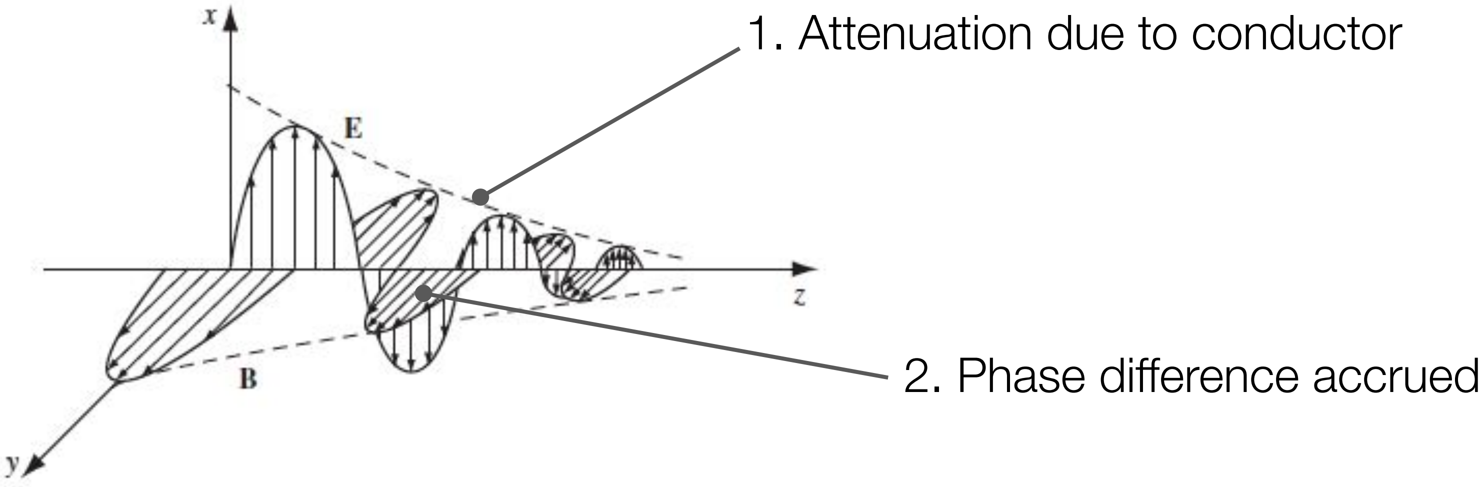


The imaginary component, due to the contribution of *conductance* and *frequency*, results in exponential decay of the fields inside the material — **Skin depth**

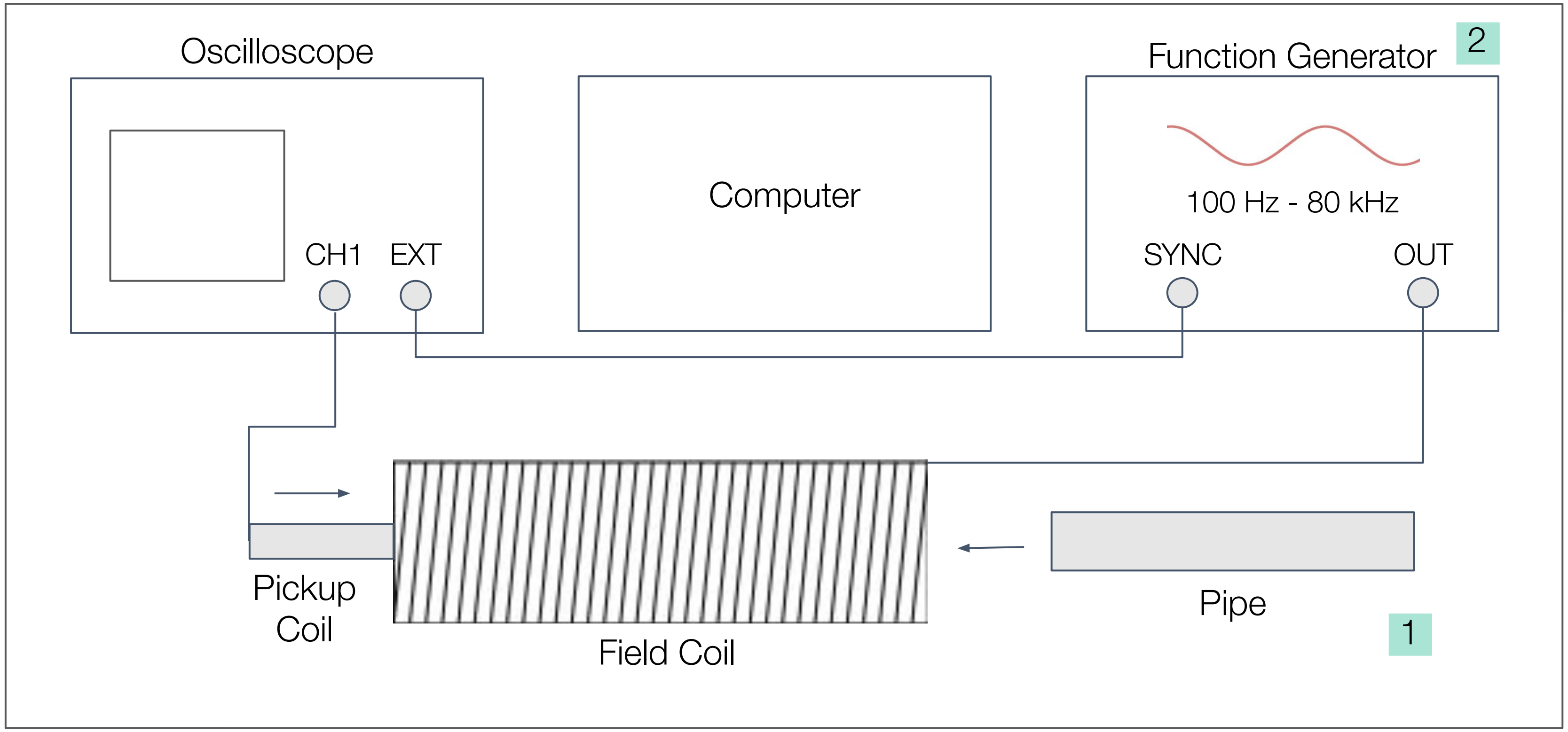
Objectives

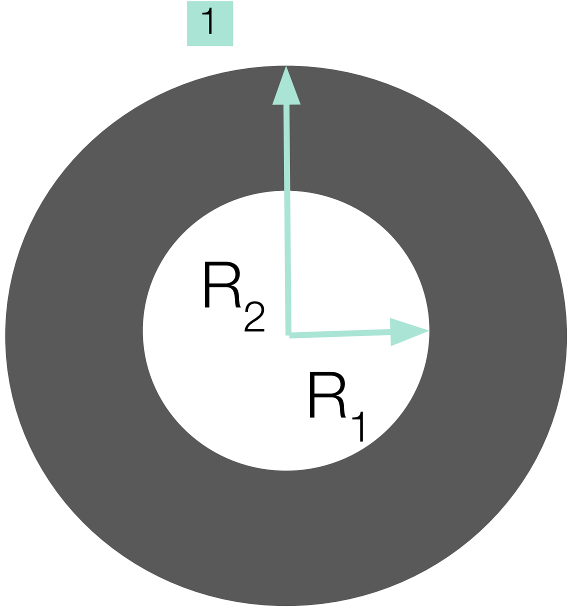
1. Utilize the phenomena of skin depth to measure the conductivity of copper (Cu) and aluminum (Al)

2. Determine the best parameter for the measurement of conductivity



Overview of Setup





Cross sectional view of conducting pipe

Analysis Model

1. Signal from pickup coil is detected by the oscilloscope

$$\frac{H_i}{H_o} = \rho e^{i\phi}$$

2. Signal is fit to a cosine to determine amplitude and phase

$$A + B\cos(\omega t - \phi)$$

3. Cosine fit is related back to Bessel function solutions

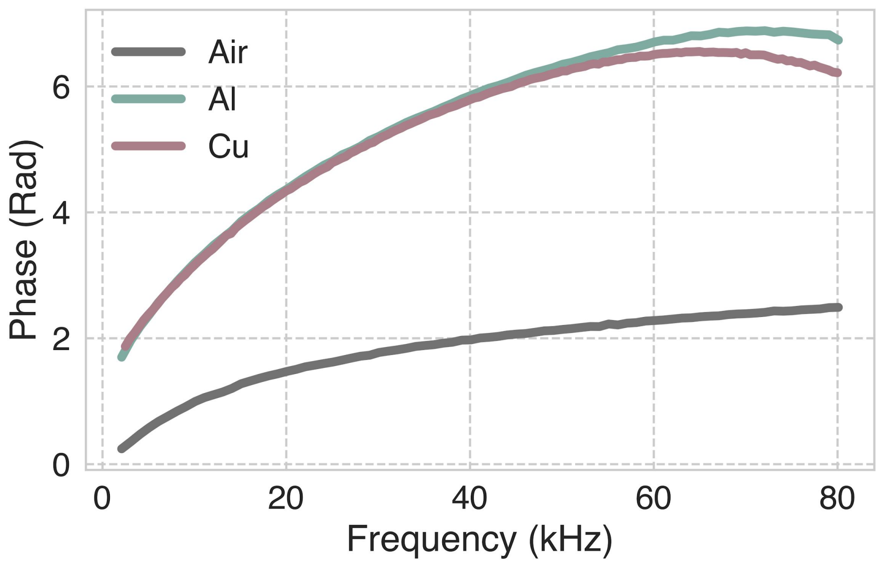
$$2\sqrt{\frac{R_2}{R_1}}\left\{\frac{e^{-k_o(R_2-R_1)}}{\sqrt{1+R_1k_o+\frac{R_1^2k_e^2}{2}}}\right\}$$

$$k_o\left(R_2-R_1\right)+\arctan\left(\frac{R_1k_o}{2+R_1k_o}\right)$$

4. Conductivity is determined through a least squares fit over frequency range

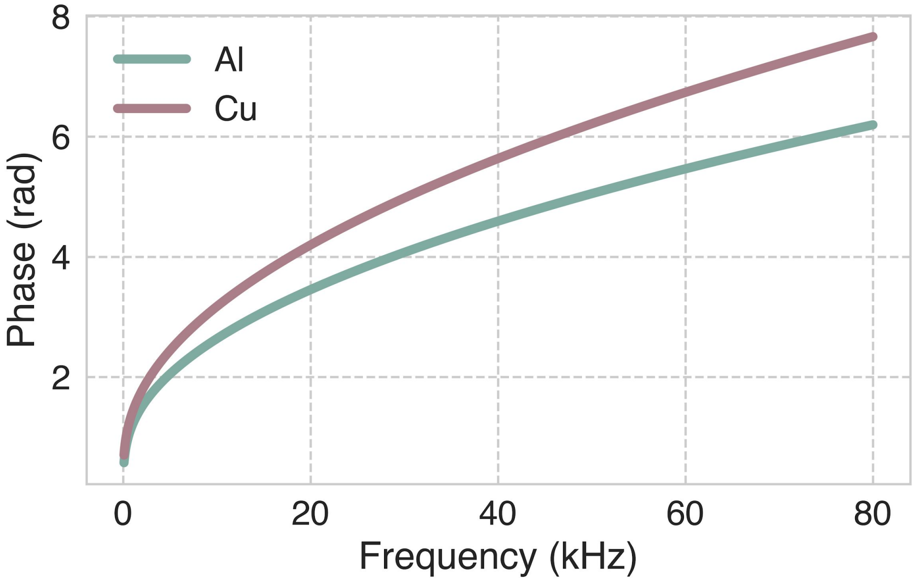
$$\boxed{k_o = \sqrt{\frac{\omega\sigma\mu}{2}}}$$

Results: Phase Extraction



Measured Values

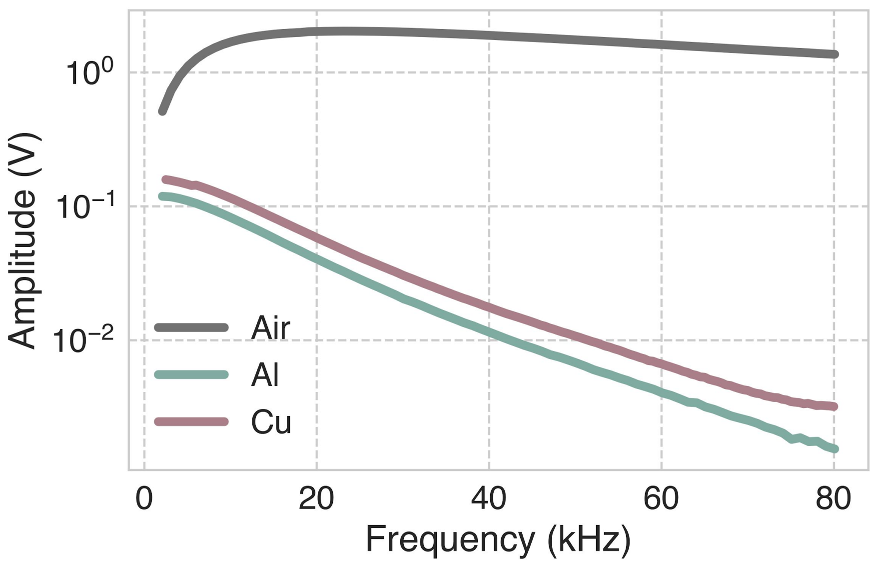
The phase fit results from the computer for data points collected at 1 kHz spacing from 100 Hz to 80 kHz



Expected Values

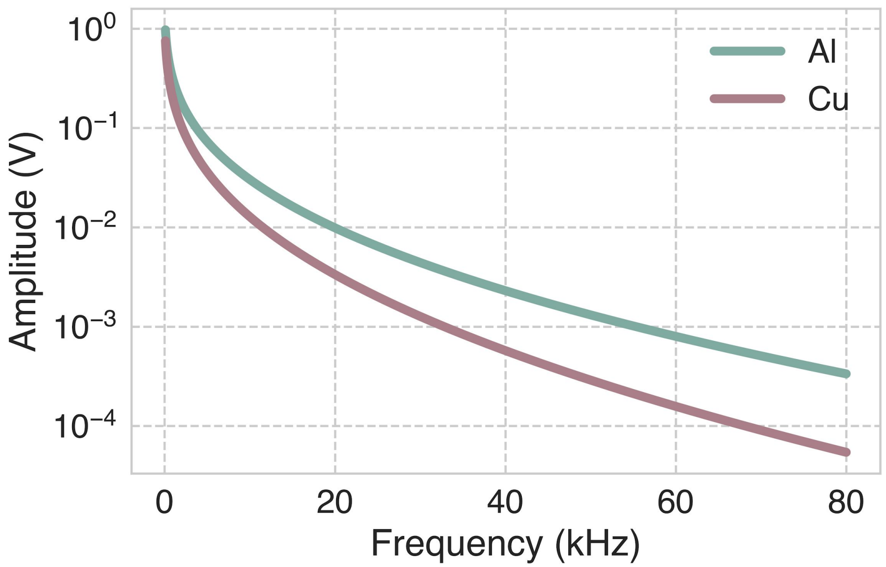
Calculated with the phase relation at frequencies with 1 kHz spacing from 100 Hz to 80 kHz

Results: Amplitude Extraction



Measured Values

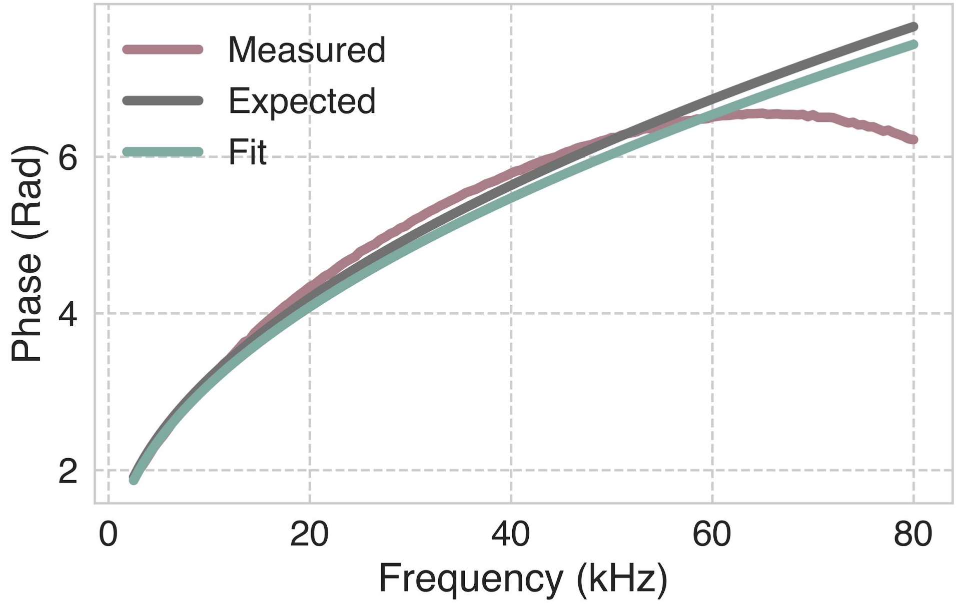
The amplitude fit results from the computer for data points collected at 1 kHz spacing from 100 Hz to 80 kHz



Expected Values

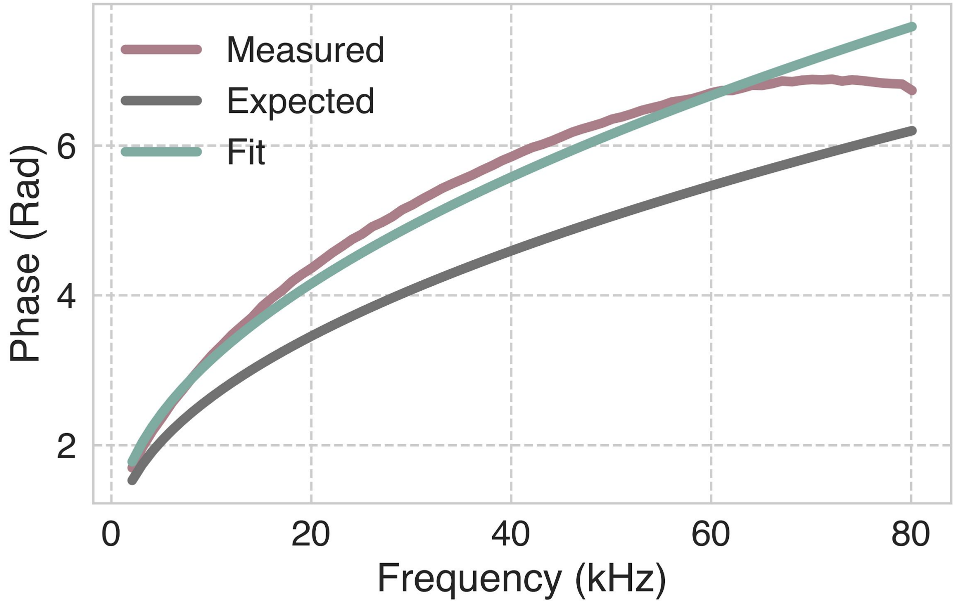
Calculated with the amplitude relation at frequencies with 1 kHz spacing from 100 Hz to 80 kHz

Results: Least Squares Fits



Copper

Conductance measured: 5.57E5 1/(Ω cm)
Expected conductance: 5.95E5 1/(Ω cm)



Aluminum

Conductance measured: 5.93E5 1/(Ω cm)
Expected conductance: 3.77E5 1/(Ω cm)

Conclusions

Promising method for a quick estimate of conductivities

Mathematical model may only be accurate at low frequencies

Contributions may be:

1. Asymmetries in setup
2. Skin effect reduction of the internal inductance of a conductor

Thank you!

Special thanks to Aaron Janz